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An application of Kozeny-Carman flow resistivity model to predict the acoustical properties of polyester fibre

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Abstract

Modelling of the acoustical properties of polyester fibre materials is usually based on variations of the Bies and Hansen empirical model [1], which allows the calculation of the air flow resistivity of a porous material. The flow resistivity is the key non-acoustical parameter which determines the ability of this kind of materials to absorb sound. The main scope of this work is to illustrate that an alternative theoretical model based on the Kozeny-Carman equation can be used to predict more accurately the flow resistivity from the fibre diameter and bulk material density data. In this paper the flow resistivity is retrieved from the acoustic absorption coefficient data for polyester fibre samples of different densities and fibre diameters. These data agree closely with the flow resistivity predicted with the proposed Kozeny-Carman model.

Keywords: Polyester Fibre, Airflrow Resistivity Modelling,
Kozeny-Carman.

1. Introduction

Polyester fibres are a good acoustic absorbing material [2]. Acoustic absorbing panels based on polyester fibres are becoming popular in building, automotive and industrial noise control applications and compete against traditional mineral fibre absorbers. As a material, polyester fibre is characterised by high porosity which is usually above $\phi \geqslant 0.97$. In many applications the polyester fibre is compressed and therefore altering the bulk material density and porosity. Although this change in material properties is usually relatively small, it can significantly affect the flow resistivity of fibres, the parameter which is largely responsible for their fundamental acoustical properties.

Some previous studies on the acoustical properties of polyester fibres relied on empirical relation between the fibre diameter, density and flow resistivity (e.g. [3],[1],[4]). The flow resistivity expression in the work by Nichols (page 871 in ref. [3]) requires an adjustable parameter $0.3 \leqslant x \leqslant 1$ which value depends on the way the fibres are distributed in the material. The flow resistivity expression in the work by Bies and Hansen is described by an empirically derived function f_1 (eq. (4) in ref. [1]). The flow resistivity expression in the work by Garai and Pompoli includes two empirical coefficients A and B which depend on the fibre type and diameter (see eq. (2) and Table 1 in ref. [4]). Other studies relied on the Johnson – Champoux – Allard model with five non-acoustical parameters (e.g. [5, 6]), which may be a rather complicated model to be used to describe the acoustical properties of a relatively simple material such as polyester fibres given the uncertainties in the bulk material density. It can be shown that the bulk material density is a dominant parameter in terms of the resultant acoustical properties and it is of interest to relate these properties unambiguously to the bulk material density which is easy to measure directly.

This paper shows that the acoustical properties of polyester fibres can be predicted with a simpler model which is based on the flow resistivity alone. It is also shown that the flow resistivity is related theoretically to the fibre diameter and bulk material density using the approach suggested by Carman and Kozeny [7, 8] in the 1930s. This approach appears more physical and provides a more accurate estimate of the flow resistivity of polyester fibre from the fibres density and fibre diameter data.

The paper is organised as follows: Section 2 describes the experimental procedure and equipment used to carry out the experiment. Section 3 presents the Miki model which was used to retrieve the airflow resistivity data. Section 4 presents the Kozeny-Carman model that was adopted to explain the airflow resistivity data retrieved from the acoustical data. In this section a comparison is made between the predicted results from Kozeny-Carman approach and the experimental results. Finally, conclusions are drawn in Section 5.

2. Experimental Methodology

This study was focused on polyester fibres which mean diameter ranging from 20.2 μm up to 39.2 μm. Figure 1 shows a magnified photograph of polyester fibres which diameter is approximately 25 μm.

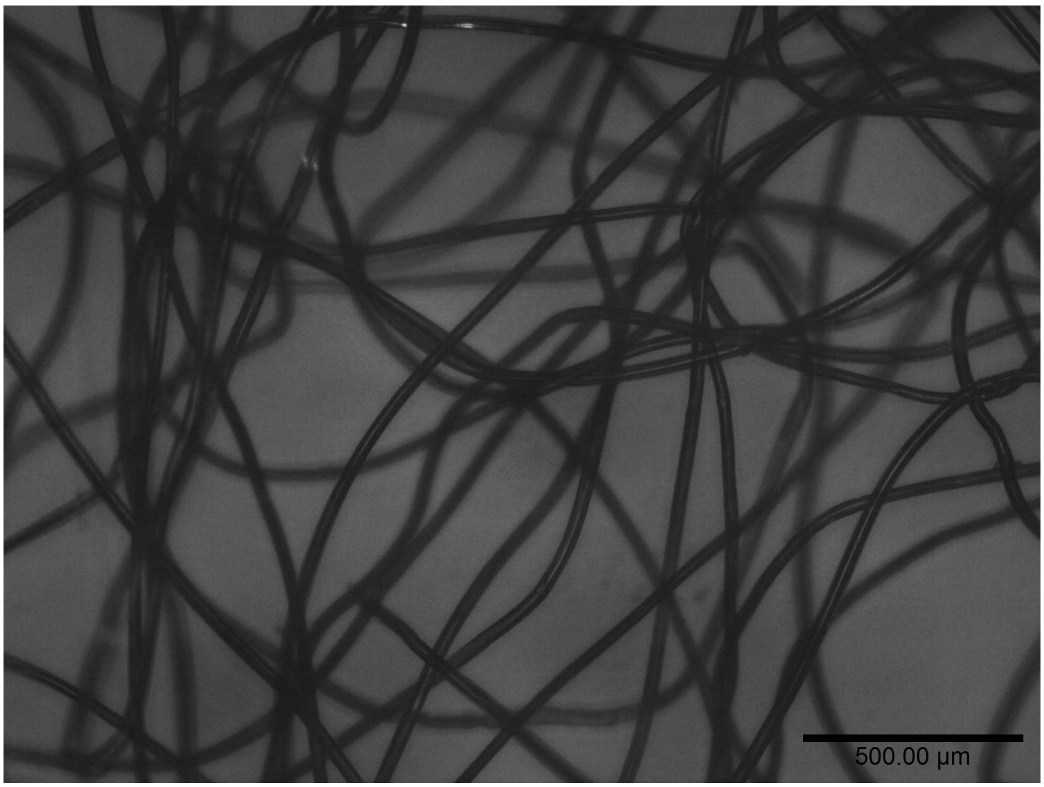


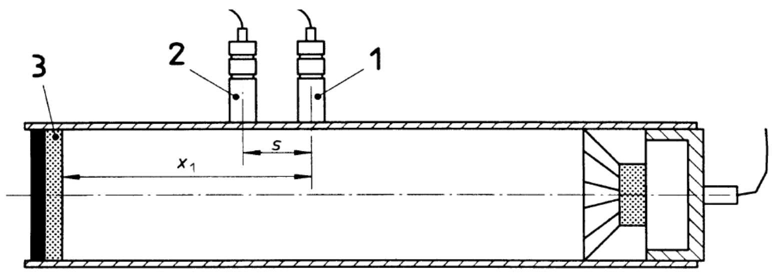
Figure 1: A photograph of polyester fibres with 24.8 µm diameter.

The textile industry tends to classify polyester fibres based on their Denier, which is a unit of measure for the linear mass density and is defined as the mass in grams per 9000 m. The relation between fibre Denier and diameter for the samples tested in this paper are given in Table 1.

Table 1: Polyester fibre denier - diameter relation

Fibre	Denier	Diameter (μm)
Sample 1	4	20.2
Sample 2	6	24.8
Sample 3	15	39.2

In modelling of the acoustical behaviour of highly porous materials, the determination of the physical parameters (airflow resistivity, open porosity, tortuosity and viscous and thermal characteristic lengths [5]) can be a tedious process. Therefore, it becomes more common to use inverse methodologies to retrieve the desired characteristic parameters [9] The work presented in this paper is based on the inversion of the flow resistivity alone from the acoustic absorption coefficient data obtained from a 2-microphone impedance tube experiment. The procedure for measuring the acoustic absorption coefficient is detailed in ref. [10]. The experiment reported in this paper involved the measurement of the absorption coefficient of polyester fibre samples under a range of bulk densities, from which the flow resistivity values are extrapolated. The bulk densities for which each sample was tested were $22.8, 24.4, 32.9$ and 40.1 kg/m^3. This measurement was carried out in a 100 mm impedance tube which is shown in Figure 2. The sample holder with an adjustable piston was used to control the density of the polyester fibre sample in the tube by adjusting the sample length l_s for a given mass of fibres in the sample followed by the acoustic measurement. In this way the acoustic absorption coefficient of a hard-back sample of polyester fibre was measured for a range of material densities and in the frequency range of 100–1600 Hz.



Key

1 Microphone A

2 Microphone B

3 Test specimen

Figure 2: Impedance tube arrangement (adapted from ref. [10]).

The flow resistivity was retrieved from the acoustic absorption coefficient data using the indirect method of parameter inversion which is based on finding the minimum of the following function:

$$F(\sigma) = \sum_n |a_e(f_n) - a_m(f_n, \sigma)| \tag{1}$$

where α_e is the experimental absorption coefficient α_m is the predicted absorption coefficient, f_n is the frequency of sound and σ is airflow resistivity. The absorption coefficient α_m was predicted using the 1-parameter Miki model [11] which predicts the characteristic impedance and complex wavenumber with the following expressions:

$$z_b(f) = \left\{1 + 0.070\left(\frac{f}{\sigma}\right)^{-0.632} + 0.107i\left(\frac{f}{\sigma}\right)^{-0.632}\right\} \tag{2}$$

and

$$k_b(f) = \frac{2\pi f}{c_0}\left\{1 + 0.109\left(\frac{f}{\sigma}\right)^{-0.0618} + 0.160i\left(\frac{f}{\sigma}\right)^{-0.618}\right\} \tag{3}$$

respectively. Here, c_0 is the speed of sound in air and $i = \sqrt{-1}$. The absorption coefficient for a hard backed specimen of polyester fibre is calculated from the following expression:

$$\alpha = 1 - \left|\frac{z_s - 1}{z_s + 1}\right|^2 \tag{4}$$

where z_s is the normalised acoustic surface impedance of a hard backed layer polyester fibre sample of thickness l_s given by:

$$z_s = z_b \coth\left(-ik_b l_s\right) \tag{5}$$

In this paper the minimisation problem defined by equation (1) was solved using the Nelder – Mead simplex (direct search) optimisation method [12]. This enabled us to determine the value of the flow resistivity which provided the best fit between the values of the absorption coefficient measured in the described experiment and those predicted by the Miki model [11]. Figure 3 illustrates the comparison between the measured absorption coefficient and that predicted with the optimised value of the flow resistivity. The proposed parameter inversion procedure the maximum mean error between the measured and predicted absorption coefficient spectra was below 0.5 %.

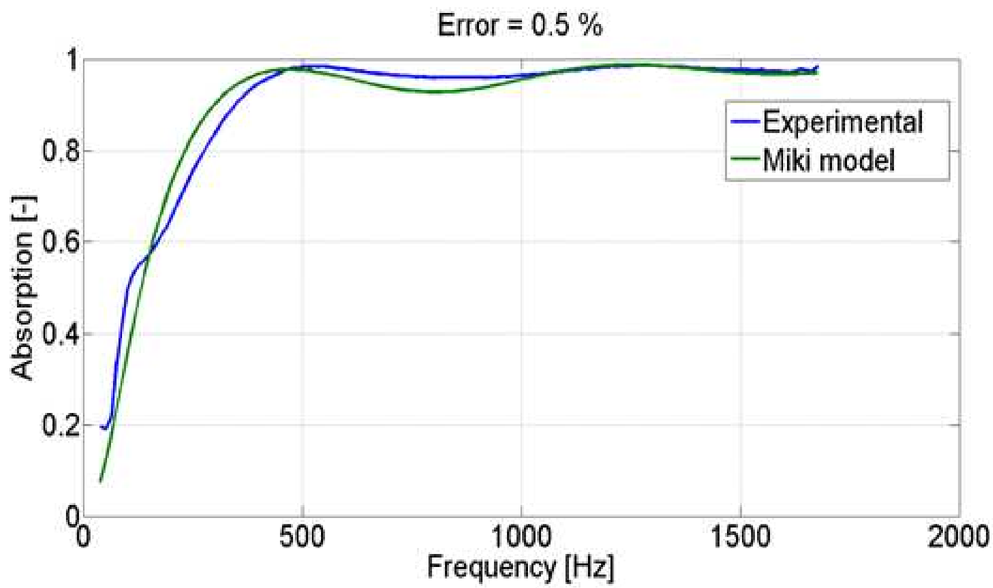


Figure 3: The measured and predicted absorption coefficient for 200 mm hard-backed layer of polyester fibres with 39.2 µm diameter and bulk density 24.4 kg/m^3

The Kozeny-Karman equation which was used in this paper was developed in the 1930s to relate the porosity of granular media, ϕ, particle size, d, and flow resistivity, σ,

$$\sigma = \frac{180\mu\left(1-\phi\right)^2}{d^2\phi^3} \tag{6}$$

It is a physical relation which can be derived from the Poiseuille's equation for laminar flow of fluid with the dynamics viscosity, μ. In the case of a medium which is composed of fibres of diameter, d_f, the particle diameter in equation (6) can be set to the fibre diameter $d \equiv d_f$. The porosity in equation (6) can be estimated from the ratio of the bulk material density, ρ_m, to the density of the polyester fibre, ρ_f, i.e.

$$\phi = 1 - \frac{\rho_m}{\rho_f} \tag{7}$$

which is a directly measurable quantity. In this study a value of 1380 kg/m^3 was adopted. If we define:

$$\gamma = \frac{\left(1-\phi\right)^2}{\phi^3} \tag{8}$$

then, according to the Kozeny-Carman equation, the airflow resistivity is a linear function of which slope depends inversely on the fibre diameter squared, d_f^2, i.e.:

$$\sigma = \frac{180\mu}{d_f^2}\gamma \tag{9}$$

3. Results

Figure 4 a comparison of all the flow resistivity values retrieved from the acoustic absorption data, σ_a, against the flow resistivity values predicted by the Kozeny-Carman model, σ_p, based on the measured values of d_f and ρ_m. The linear fit $\sigma_p = a\sigma_a$ with $a = 0.971$ shown on Figure 4 suggests that the dependence between the two data sets is linear with the coefficient of determination of $R^2 = 0.981$ and that the mean error between the two data sets is within 3%. The maximum relative error between the flow resistivity retrieved acoustically and that predicted using the Kozeny-Carman model is 10% in the case of sample 2 with the density of $\rho_m = 24.3$ kg/m^3.

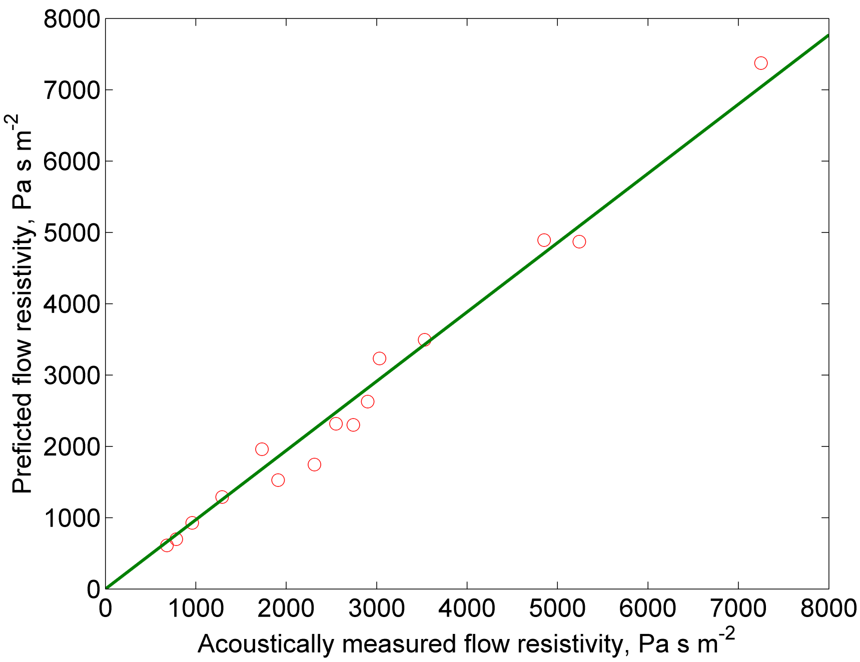


Figure 4: The correlation between the flow resistivity retrieved from the acoustical data and flow resistivity predicted with expression (6).

The ratio $\varepsilon = \rho_m/\rho_f$ for the values of ρ_m and ρ_f considered in this work is small relative to 1 and it does not exceed $\varepsilon \simeq 0.03$ for $\rho_m = 40.1$ kg/m^3. In this case the expression for γ can be approximated with $\gamma \simeq (\rho_m/\rho_f)^2$, which suggests that the flow resistivity of polyester fibre increases proportionally to the squared bulk material density. In this case, the following approximation can be suggested to estimate the flow resistivity of polyester fibre from the knowledge of the fibre diameter, d_f, and bulk material density, ρ_m:

$$\sigma \simeq \frac{180\mu}{\rho_f^2}(\rho_m/d_f)^2 \tag{10}$$

or following the notations used for the Garai-Pompoli model [4] it is possible to suggest that:

$$\sigma \simeq A\rho_m^B, \tag{11}$$

where $A = K_2/d_f^2$, $K_2 = 180\mu/\rho_f^2$ ($K_2 = 1.736 \times 10^{-9}$ for air at 20^oC and $\rho_f = 1380$ kg/m^3) and $B = 2$.

Equation (11) can be considered as a physical link between the flow resistivity, fibre diameter and bulk material density than empirical relation (4) in ref. [1] or equation (2) in ref. [4] because it is based on the expressions derived from the Poiseuille's equation for laminar flow [7, 8]. For the range of densities adopted in this work approximation (10) is accurate within 10%, which is sufficient for the prediction of the acoustical properties of polyester fibre which bulk density can vary by a similar amount because of the uncertainties in the degree of fibre compression, fibre density, fibre diameter and acoustical data.

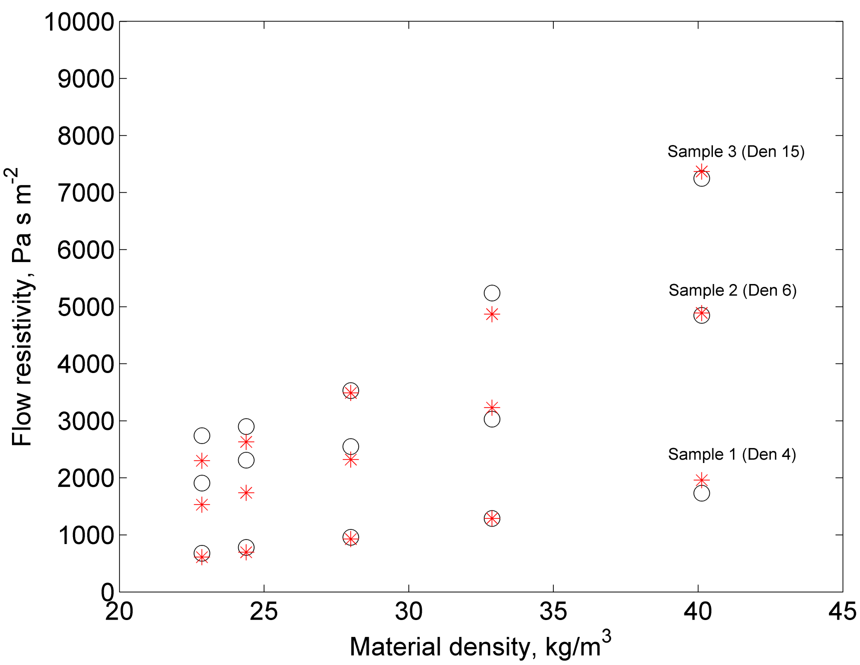


Figure 5: The flow resistivity as a function of bulk density. (∘) - the flow resistivity retrieved from the acoustical data; (*) - the flow resistivity predicted with expression (6).

4. Conclusions

This work has demonstrated that a relatively simple 1-parameter model based on the air flow resistivity data only is sufficient to predict accurately (maximum mean error in the predicted absorption coefficient less than 0.5%) the acoustical properties of polyester fibre with a relatively low bulk material density ($22.9 \leq \rho_m \leq 40.1$ kg/m^3). It has also been demonstrated that this model can be used to retrieve the air flow resistivity of polyester fibre from acoustic absorption coefficient data measured in a standard impedance tube. It has been shown that the values of the flow resistivity retrieved from the acoustical absorption data agree within 10% with the flow resistivity predicted with the Kozeny-Carman model (equation (6)). Unlike some other models which can be used to predict the flow resistivity of polyester fibre, the proposed Kozeny-Carman model is derived from the Poiseuille's equation for laminar flow and presents a theoretical links between the flow resistivity, fibre diameter and bulk material density. The maximum error between the predicted and acoustically retrieved flow resistivities has been found to be 10% in the case of sample 2 with the bulk material density of 24.3 kg/m^3. An equation, alternative to the empirical Garai-Pompoli model [4] has been suggested (equation (11). In this equation the coefficients A and B which are empirically defined in the Garai-Pompoli model have been explained theoretically. It has been shown experimentally and theoretically that the flow resistivity of polyester fibre increases proportionally with the squared value of the bulk material density, ρ_m^2, and decreases proportionally with the squared value of the fibre diameter, d_f^2.

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